

STAT 689: Constrained Estimation and Inference
SPRING 2022

Statistical data analysis often involves estimation of a function. Examples include regression functions, probability curves, density and hazard functions, and variance functions. Estimation of these functions is based on a statistical model constructed from a set of assumptions, which ideally are backed by theory or *a priori* information specific to the situation. Often the *valid* assumptions are vague or qualitative in nature. A regression function might be known to be smooth, a probability curve to be increasing in dose, a hazard function to be convex, or a density might be known to be unimodal. Incorporating assumptions based on shape or ordering may be accomplished through inequality constraints. The main topics of this course are estimation and statistical inference using models incorporating these constraints.

We will begin the course with motivational examples of various situations in which constrained estimation is appropriate. Next, we will review the basics of linear models theory as appropriate for the material in this course. The review of linear models might take two class periods. We will emphasize that the constrained regression model can be thought of as a mixture of linear regression models. Nonparametric function estimation using regression splines will be covered prior to the material on constrained regression splines.

The R packages `coneproj` and `cgam` will be used to write code and analyze data.

Tentative List of Topics in Constrained Estimation and Inference:

1. Examples of Statistical Models using Linear Inequality Constraints
2. Extended Review of Linear Models Theory
3. Edges and Faces of a Polyhedral Convex Cone
4. Characterization of Least-Squares Estimation under Inequality Constraints – The KKT Conditions
5. The Polar Cone
6. More Constraints than Dimensions
7. Cone Projection Algorithm

8. Quadratic Programming, Weighted Least-Squares, and Constrained Parametric Regression
9. Inference Methods in Constrained Parametric Regression
10. Testing for a Linear Inequality Constraint
11. Nonparametric Regression with Splines; Penalized Splines
12. Constrained Spline Regression
13. Constrained Penalized Spline Regression
14. Partial Linear Model, Inference
15. Generalized Additive Model with Constrained Splines
16. Variable and Shape Selection

Class meeting time: M 4:30-7:10

Office hours: TBA

Grading:

- Homework: 80%.
- Project and presentation: 20%.